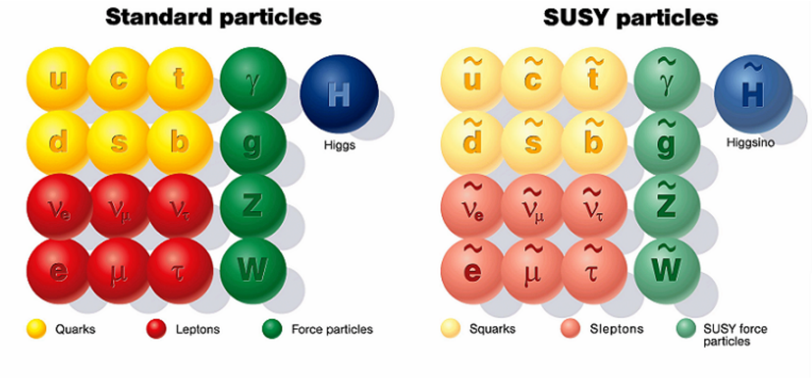
institut für theoretische physik

Graduiertenkolleg 2149 Research Training Group

Ward identities in $\mathcal{N}=1$ supersymmetric SU(3) Yang-Mills theory on the lattice

Sajid Ali

GRK 2149 Annual Retreat, 25-28.09.2017


Standard particles
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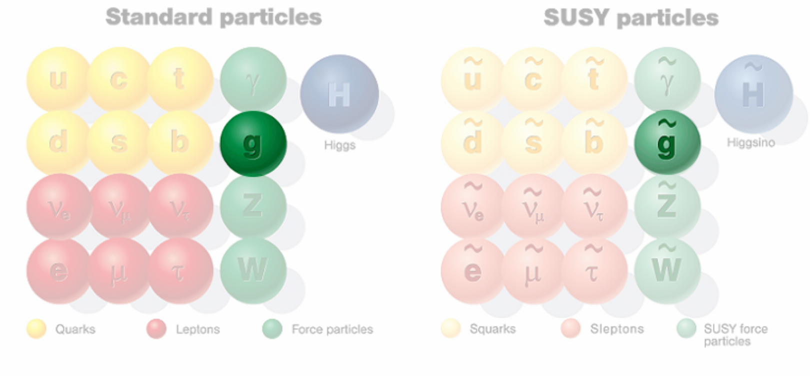
SUSY force particles



$\mathcal{N}=1$ supersymmetric Yang-Mills theory

Supersymmetric Yang-Mills theory

$$S=\int d^4x\left\{-\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}+\frac{\mathrm{i}}{2}\bar{\lambda}^a\gamma_\mu(\mathscr{D}_\mu\lambda)^a-\frac{m_g}{2}\bar{\lambda}^a\lambda^a\right\}$$

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- Field strength tensor

$$F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu-\mathrm{i}g\,f^a_{bc}[A^b_\mu,A^c_\nu]$$

- Covariant derivative in adjoint representation

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- Vector supermultiplet

1) Gauge field(gluon) A^a_μ
2) Majorana-spinor field(gluino) λ^a, $\quad \bar{\lambda}^a = \lambda^{aT} C$

Supersymmetric Yang-Mills theory

- SUSY transformations (on-shell):

$$\delta A_\mu^a = -2g\bar{\lambda}^a\gamma_\mu\varepsilon\,,$$
$$\delta\lambda^a = -\frac{\mathrm{i}}{g}\sigma_{\mu\nu}F_{\mu\nu}^a\varepsilon$$

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- In contrast to QCD:
  1) λ^a is Majorana spinor field, "$N_f = \frac{1}{2}$"
  2) adjoint representation of $SU(N_c)$
- Gluino mass term $\frac{m_g}{2}\bar{\lambda}^a\lambda^a$ breaks SUSY softly

Motivation

- SYM: simplest model with SUSY and local gauge invariance
- Part of the supersymmetrically extended Standard Model
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- Part of the supersymmetrically extended Standard Model
- Possible connection to ordinary QCD
- Similar to QCD:
  1) Asymptotic freedom
  2) Confinement
  3) Numerical lattice simulation of bound states

Motivation

Solution of non-perturbative problems:

- Spontaneous breaking of chiral symmetry $Z_{2N_c} \to Z_2$ $\longleftrightarrow$ Gluino condensate: $< \lambda\lambda > \neq 0$
- Spectrum of bound states $\to$ Supermultiplets

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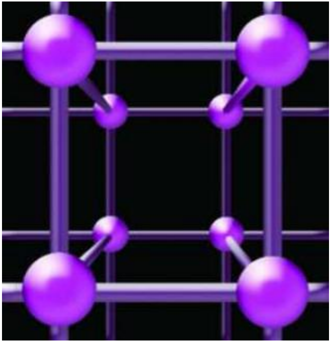

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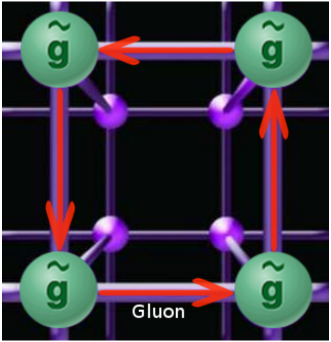

Link:
$$U_{x\mu} = e^{iagA_\mu(x)}$$

Plaquette:
$$U_{x\mu\nu} = e^{ia^2 F_{\mu\nu}(x)}$$

Supersymmetric Yang-Mills theory on the lattice

$$S = -\frac{\beta}{N_c}\sum_p \mathrm{Re}\,\mathrm{Tr}\;U_p$$

$$+\frac{1}{2}\sum_x\left\{\overline{\lambda}^a_x\lambda^a_x - \kappa\sum_{\mu=1}^{4}\left[\overline{\lambda}^a_{x+\hat{\mu}}V_{ab,x\mu}(1+\gamma_\mu)\lambda^b_x + \overline{\lambda}^a_x V^t_{ab,x\mu}(1-\gamma_\mu)\lambda^b_{x+\hat{\mu}}\right]\right\}$$

$\beta = \frac{2N_c}{g^2}$, $\kappa = \frac{1}{2m_0+8}$(hopping parameter), m_0 : bare gluino mass

$V_{ab,x\mu} = 2\,\mathrm{Tr}(U^\dagger_{x\mu}T_aU_{x\mu}T_b)$, adjoint link variables

Supermultiplets

Colour neutral bound states of gluons and gluinos

Predictions from effective Lagrangeans:

Chiral supermultiplet (Veneziano, Yankielowicz)

- 0^- gluinoball a - η' $\sim$ $\bar{\lambda}\gamma_5\lambda$
- 0^+ gluinoball a - f_0 $\sim$ $\bar{\lambda}\lambda$
- spin $\frac{1}{2}$ gluino-glueball $\sim$ $\sigma_{\mu\nu}\,\mathrm{Tr}(F_{\mu\nu}\lambda)$

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possible mixing and Baryons

Expression in the continuum

Noether Theorem in classical theory → WIs in quantum theory

$$\left\langle \left(\partial_\mu j^\mu(x)\right) Q(y) \right\rangle = -\left\langle \frac{\delta Q(y)}{\delta \bar{\varepsilon}(x)} \right\rangle$$

RHS of equation is contact term, which is zero if Q is localized at space-time points different from x.

SUSY transformations on the lattice with P, T, Majorana nature and gauge invariance:

$$\delta U_\mu(x) = -\frac{iga}{2}\Big(\bar\varepsilon(x)\gamma_\mu U_\mu(x)\lambda(x) + \bar\varepsilon(x+\hat\mu)\gamma_\mu\lambda(x+\hat\mu)U_\mu(x)\Big)$$

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Above transformations result in following Ward identities:

$$\left\langle\left(\nabla_\mu S_\mu(x)\right)Q(y)\right\rangle = m_0\left\langle D_S(x)Q(y)\right\rangle + \left\langle X(x)Q(y)\right\rangle - \left\langle\frac{\delta Q(y)}{\delta\bar{\varepsilon}(x)}\right\rangle$$

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$D_S(x)$: due to bare gluino mass (m_0)
$X(x)$: due to lattice regularization

} *Break SUSY*

After renormalization WI gets the following form:

$$\left\langle \left(\nabla_\mu S_\mu(x)\right) Q(y)\right\rangle + \frac{Z_T}{Z_S}\left\langle \left(\nabla_\mu T_\mu(x)\right) Q(y)\right\rangle = \frac{m_S}{Z_S}\left\langle D_S(x) Q(y)\right\rangle + O(a)$$

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Zero spatial momentum WI and expansion in a basis of 16 Dirac matrices; the surviving contributions form a set of two non-trivial independent equations:

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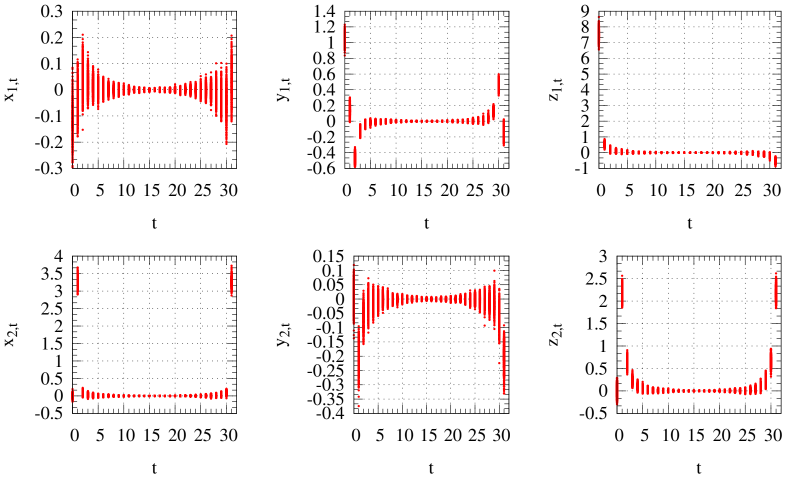
$\Longrightarrow$

$$\sum_\alpha A_\alpha x_{i\alpha} = 0, \quad i = (b,t), \alpha = 1,2,3$$

Supercomputer

Numerical results

Figure: $V = 16^3 \cdot 32,\ \beta = 5.5,\ \kappa = 0.1673$



Global method

Calculate A and B by minimizing the quantity:

$$\sum_{b=1}^{2}\sum_{t=t_{min}}^{t_{max}}\frac{(x_{b,t}+Ay_{b,t}-Bz_{b,t})^2}{\sigma^2}$$

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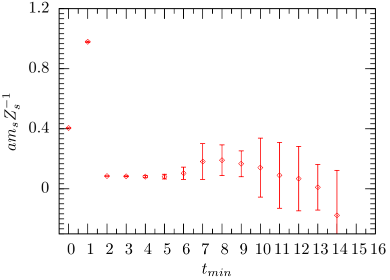

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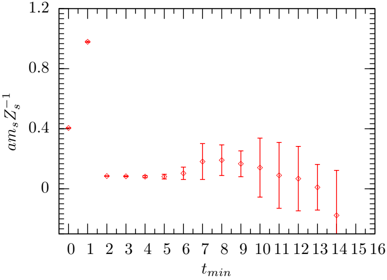
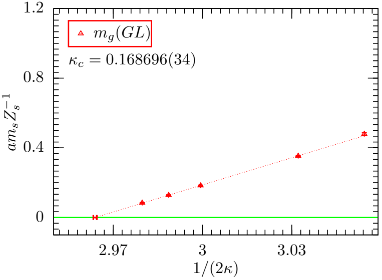


Extrapolation to the Chiral limit using m_g



Generalized χ^2 method

$$\sum_\alpha A_\alpha x_{i\alpha} = 0$$

We employ the method of
maximum likelihood

$$L = \frac{1}{2} \sum_{i,\alpha,j,\beta} (A_\alpha \bar{x}_{i\alpha})(D^{-1})_{ij}(A_\beta \bar{x}_{j\beta})$$

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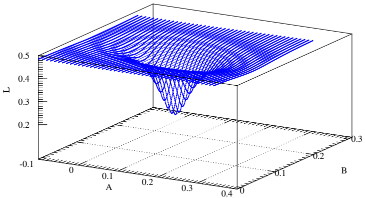

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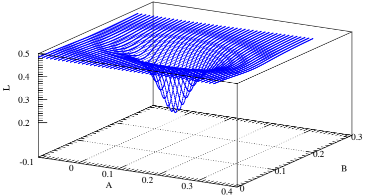

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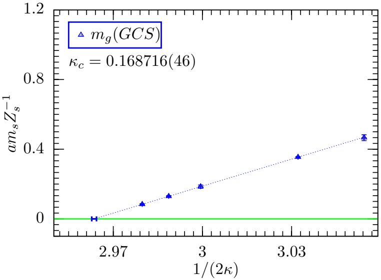

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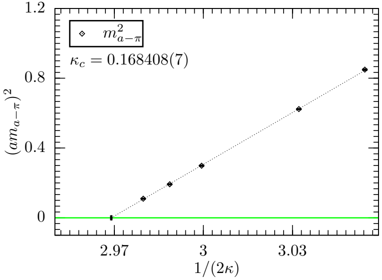

Figure: Extrapolation to the critical point /chiral limit using $m^2_{a-\pi}$

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- It is being used for the tuning of critical point

$$(am_{a-\pi})^2 \simeq A(\frac{1}{\kappa} - \frac{1}{\kappa_c})$$

$$am_S Z_S^{-1} = \frac{1}{2}(\frac{1}{\kappa} - \frac{1}{\kappa_c})$$

$$(am_{a-\pi})^2 \propto am_S Z_S^{-1}$$

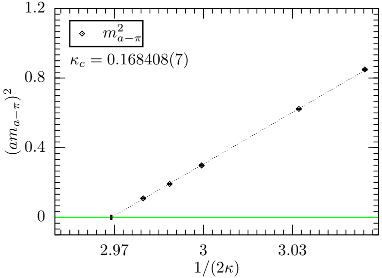

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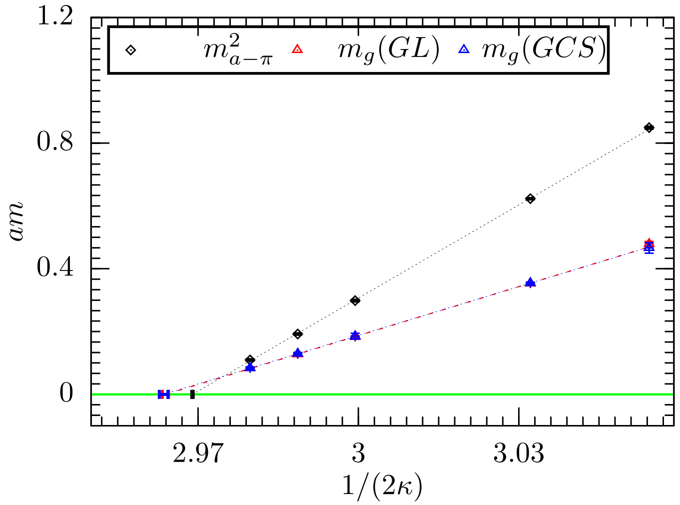
Figure: Extrapolation to Chiral limit using $m^2_{a-\pi}$ and m_g



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- Extrapolations towards vanishing gluino mass using $m_{a-\pi}^2$
- Consistency between κ_c from WIs and from $m_{a-\pi}^2$
- Consistency with restoration of SUSY

Thank You!

Figure: Schlossgebäude: Westfälische Wilhelms-Universität Münster.