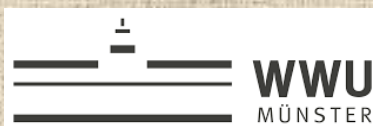


# *DARK MATTER PRODUCTION MECHANISMS*



*Jose A. R. Cembranos*



*Lectures on Dark Matter  
Jose A. R. Cembranos*

# *Dark Matter Production*

- **Standard production:**

- ▶ Freeze-out mechanism

## **Weakly Interacting Massive Particles:**

### **Thermal production**

- **Alternatives:**

- ▶ Freeze-in mechanism
- ▶ Decays of other particles
- ▶ Gravitational production
- ▶ Misalignment mechanism
- ▶ Spontaneous symmetry breaking
- ▶ Asymmetric DM
- ▶ ...

# WIMP Relic Density

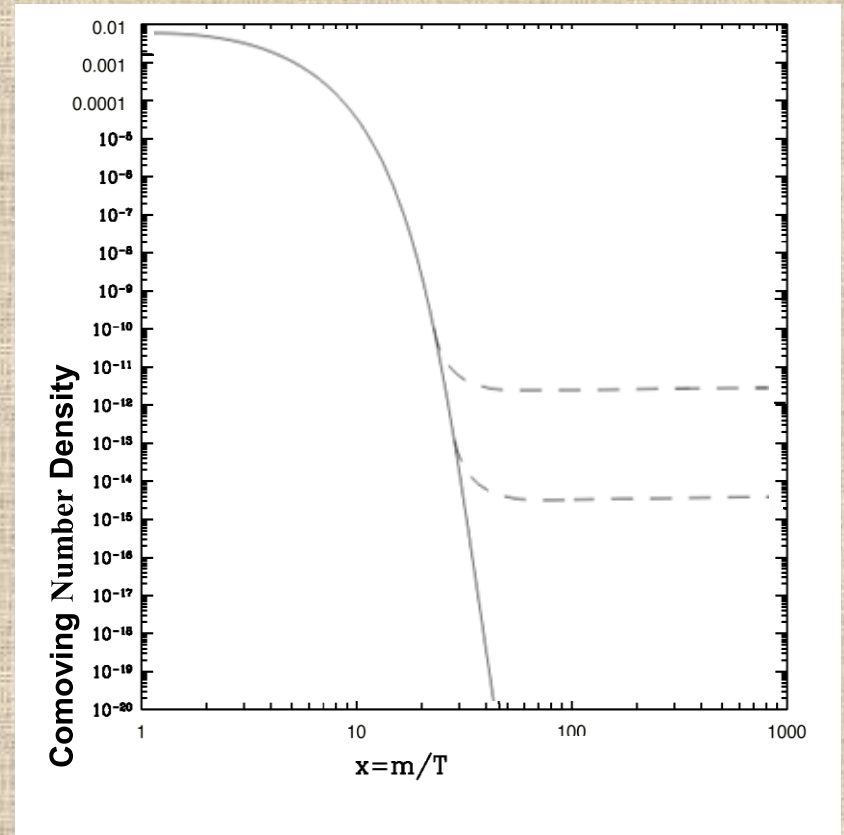
The evolution of the number density follow the Boltzmann equation:

$$dn_{\text{WIMP}}/dt = -3Hn_{\text{WIMP}} - \langle \sigma_A v \rangle [(n_{\text{WIMP}})^2 - (n_{\text{WIMP}}^{\text{eq}})^2]$$

Thermal equilibrium density:

$$n_{\text{DM}}^{\text{eq}} = g/(2\pi)^3 \int f(p) d^3p$$

When  $\Gamma = \langle \sigma_A v \rangle n_{\text{DM}} < H$ , the DM is frozen out.



# WIMP Relic Density

The evolution of the number density follow the Boltzmann equation:

$$\frac{dn_\alpha}{dt} = -3Hn_\alpha - \langle\sigma_A v\rangle(n_\alpha^2 - (n_\alpha^{eq})^2)$$

where the thermal average

$$\langle\sigma_A v\rangle = \frac{1}{n_{eq}^2} \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} f(E_1)f(E_2) \frac{w(s)}{E_1 E_2}$$

of velocity times the total annihilation cross section:

$$\sigma_A = \sum_X \sigma(\pi^\alpha \pi^\alpha \rightarrow X)$$

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$w(s)$  is defined as

$$w(s) = E_1E_2\sigma_{Av_{rel}} = \frac{s\sigma_A}{2} \sqrt{1 - \frac{4M^2}{s}}$$

with the the invariant Mandelstam variable:

$$s = (p_1 + p_2)^2 = 2(M^2 + E_1E_2 - |\vec{p}_1||\vec{p}_2|\cos\theta)$$

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The distribution functions are assumed to be thermal:

$$f(E) = \frac{1}{e^{E/T} + a}$$

and defined the equilibrium number densities:

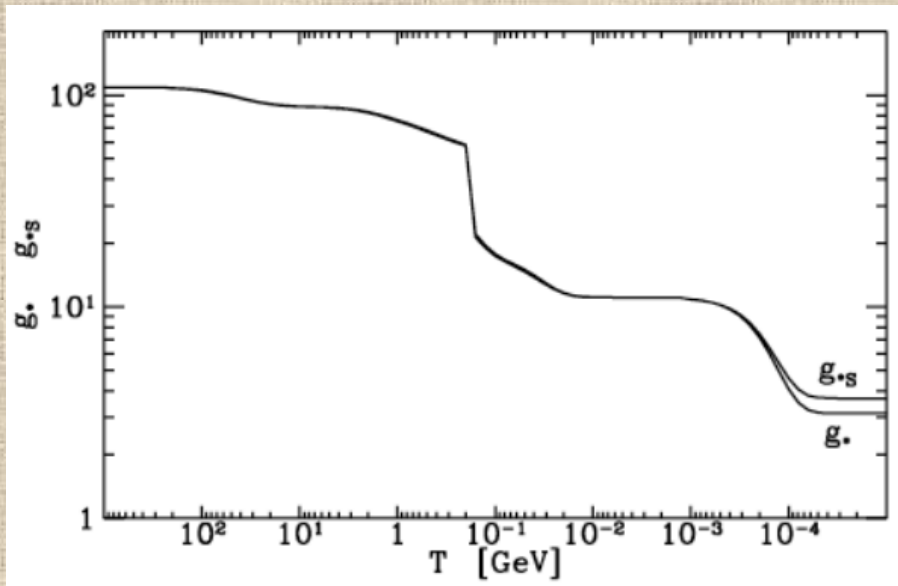
$$n_{eq} = \int \frac{d^3p}{(2\pi)^3} f(E)$$

# WIMP Relic Density

The expansion of the universe is driven by the Friedmann equation:

$$H^2 = \frac{8\pi}{3M_P^2} \rho$$

It is interesting to parameterize the energy (and entropy) content by the Effective number of relativistic degrees of freedom:



$$\rho = g_{eff}(T) \frac{\pi^2}{30} T^4$$

$$s = h_{eff}(T) \frac{2\pi^2}{45} T^3$$

They run from  
106.75 to 3.36 (energy) or  
106.75 to 3.91 (entropy)

# WIMP Relic Density

By rewriting the Boltzmann equation for the new variables:

$$x = M/T \text{ and } Y = n/s$$

we can write

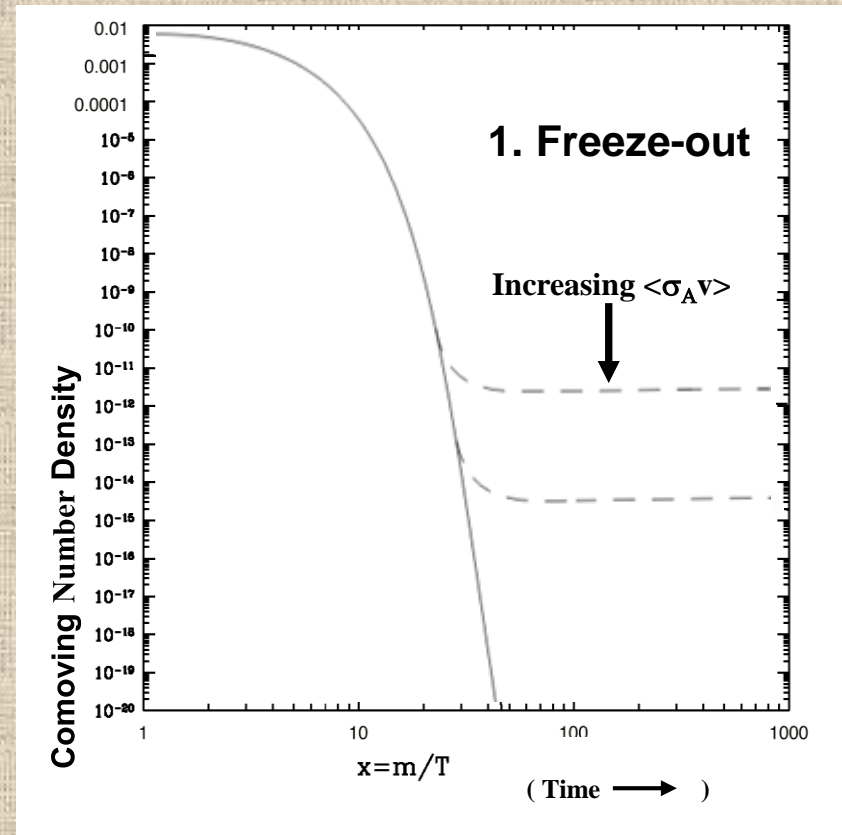
$$\frac{dY}{dx} = - \left( \frac{\pi M_P^2}{45} \right)^{1/2} \frac{h_{eff} M}{g_{eff}^{1/2} x^2} \langle \sigma_{Av} \rangle (Y^2 - Y_{eq}^2)$$

The solutions can be approximated by the thermal distribution at freeze-out:

$$x = x_f$$

$$Y_{eq}(x) = \frac{45 \zeta(3)}{2\pi^4} \frac{1}{h_{eff}(x)}, \quad (x \ll 3)$$

$$Y_{eq}(x) = \frac{45}{2\pi^4} \left( \frac{\pi}{8} \right)^{1/2} x^{3/2} \frac{1}{h_{eff}(x)} e^{-x}, \quad (x \gg 3)$$





# WIMP Relic Density

$$x = M/T \text{ and } Y = n/s$$

Freeze-out normalized number densities:

$$Y_{eq}(x) = \frac{45\zeta(3)}{2\pi^4} \frac{1}{h_{eff}(x)}, \quad (x \ll 3)$$

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Hot relics:

Estimated freeze-out temperature:

$$H(T_f) = 1.67 g_{eff}^{1/2}(T_f) \frac{T_f^2}{M_P} = \Gamma_A(T_f)$$

$$\Gamma = \langle \sigma_A v \rangle n_{DM}$$

Result:

$$\Omega_{Br} h^2 = 7.83 \cdot 10^{-2} \frac{1}{h_{eff}(x_f)} \frac{M}{\text{eV}}$$

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Hot relics:

Semi-analytical estimated freeze-out temperature:

$$x_f = \ln \left( \frac{0.038 c(c+2) M_P M \langle \sigma_{Av} \rangle}{g_{eff}^{1/2} x_f^{1/2}} \right)$$

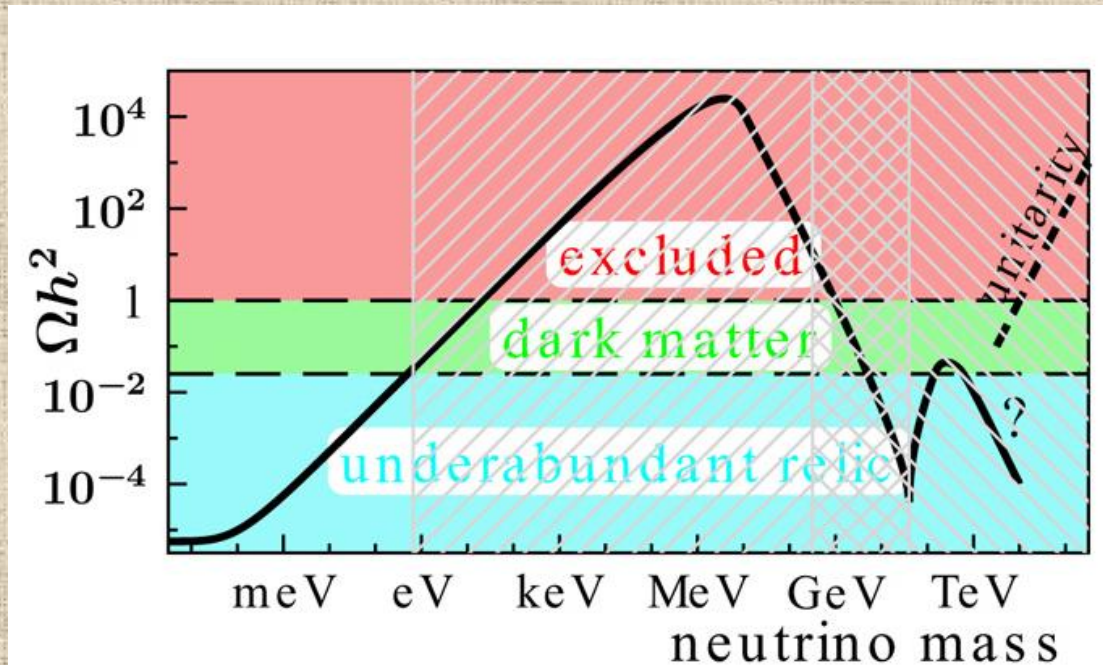
$$\langle \sigma_{Av} \rangle = \sum_{n=0}^{\infty} c_n x^{-n}$$

Result:

$$\Omega_{Br} h^2 = 8.77 \cdot 10^{-11} \text{GeV}^{-2} \frac{x_f}{g_{eff}^{1/2}} \left( \sum_{n=0}^{\infty} \frac{c_n}{n+1} x_f^{-n} \right)^{-1}$$

# WIMP Relic Density

Standard active neutrino example:



# Freeze-in Relic Density

The evolution of the number density follow the Boltzmann equation:

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where the thermal average

$$\langle\sigma_{Av}\rangle = \frac{1}{n_{eq}^2} \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} f(E_1)f(E_2) \frac{w(s)}{E_1E_2}$$

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By assuming the standard inflation scenario:

$$\begin{aligned}\frac{d\rho_\phi}{dt} &= -3H\rho_\phi - \Gamma_\phi\rho_\phi \\ \frac{d\rho_R}{dt} &= -4H\rho_R + \Gamma_\phi\rho_\phi + \langle\sigma v\rangle 2\langle E_X \rangle \left[ n_X^2 - (n_X^{eq})^2 \right] \\ \frac{dn_X}{dt} &= -3Hn_X - \langle\sigma v\rangle \left[ n_X^2 - (n_X^{eq})^2 \right] .\end{aligned}$$

Guiduce, Kolb, Riotto, arXiv:hep-ph/0005123

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$$\left(\frac{H}{H_R}\right)^2 = \left(\frac{T}{T_R}\right)^8 = \left(\frac{a}{a_R}\right)^{-3}$$

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$\Gamma_\phi = \sqrt{\frac{4\pi^3 g_*(T_{RH})}{45}} \frac{T_{RH}^2}{M_{Pl}}$

Guiduce, Kolb, Riotto, arXiv:hep-ph/0005123

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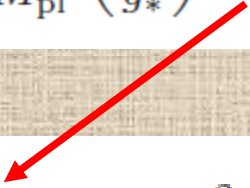
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$$\Omega_0 h^2 \simeq \frac{s_0 g^2 x_R^{-7}}{36\pi^6 H_0^2 M_{\text{pl}}} \left(\frac{90}{g_*}\right)^{\frac{3}{2}} \mathcal{F}(x_{\text{max}}),$$


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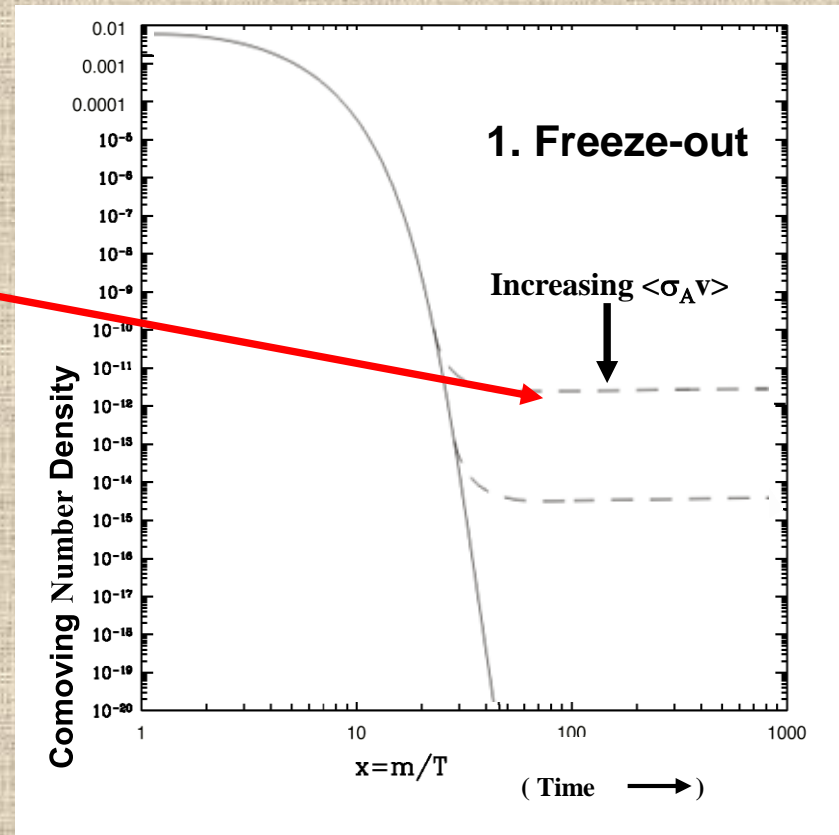
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WIMP relic density:

$$\Omega_{\text{WIMP}} h^2 \propto 1 / \langle \sigma_A v_{\text{WIMP}} \rangle$$

$$T_{\text{Freeze out}} \sim m_{\text{WIMP}} / 20$$



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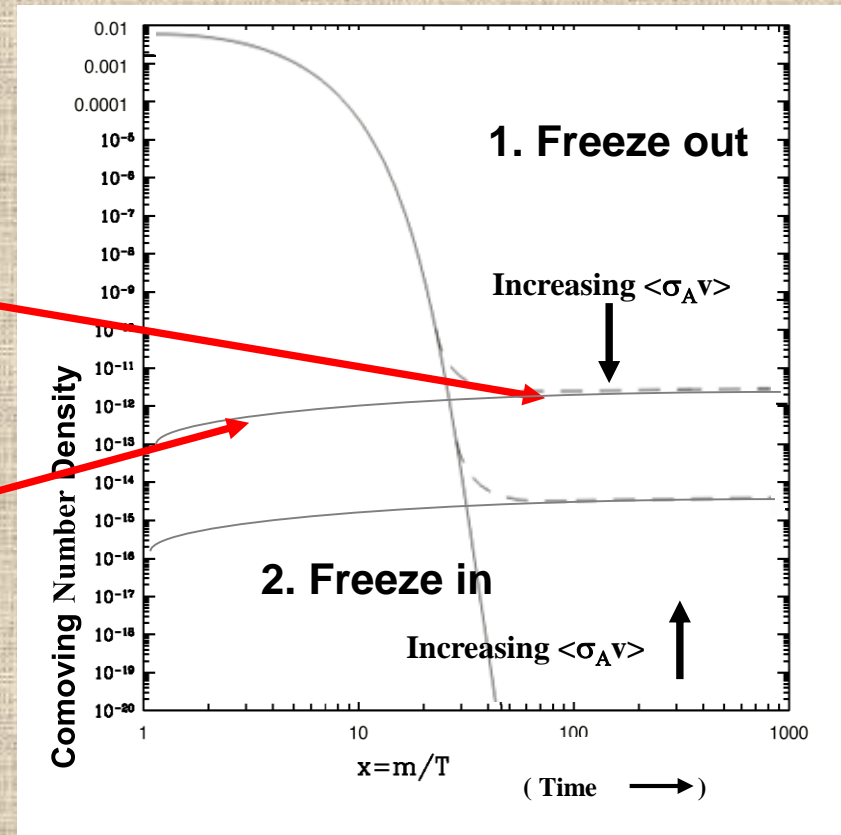
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Freeze-in relic density:

$$\Omega_{\text{FI}} h^2 \propto \langle \sigma_A v_{\text{DMI}} \rangle$$



# Decays of other particles

The evolution of the number density follow the Boltzmann equation:

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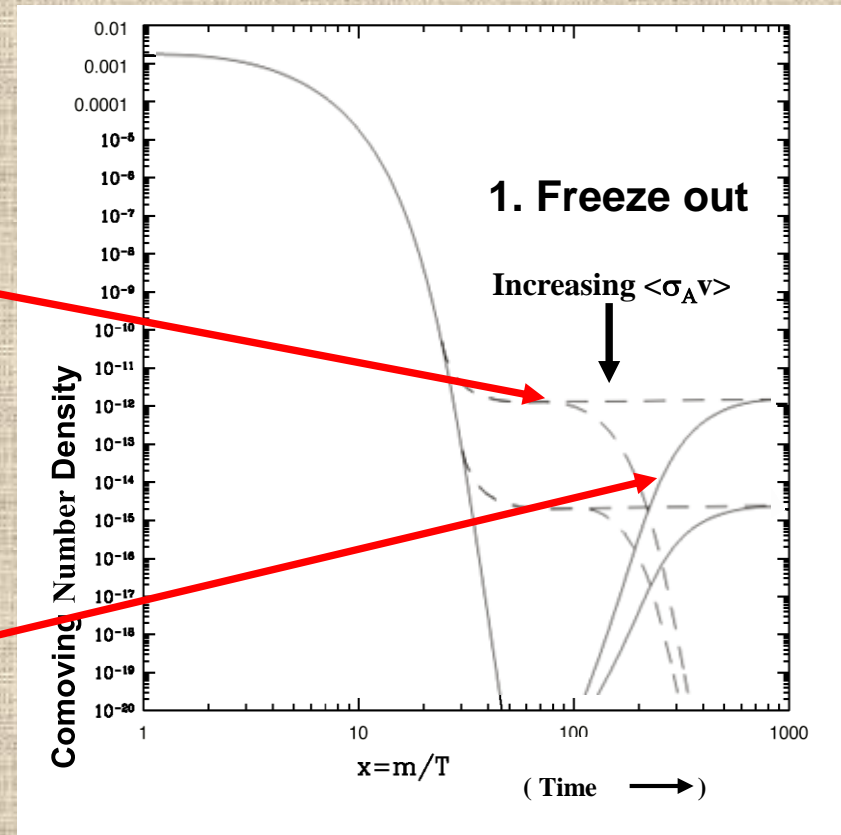
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SWIMP relic density:

$$\Omega_{\text{SWIMP}} h^2 = \Omega_{\text{WIMP}} h^2 m_{\text{SWIMP}} / m_{\text{WIMP}}$$

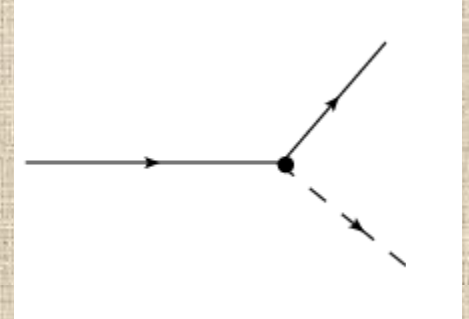
$$\propto 1/\langle \sigma_A v_{\text{WIMP}} \rangle (m_{\text{SWIMP}} / m_{\text{WIMP}})$$



# Gravitational decays

Planck scale suppressed decay:

$$\tau \simeq \frac{3\pi}{b} \frac{M_P^2}{(\Delta m)^3} \simeq \frac{3.57 \times 10^{22} \text{ s}}{b} \left[ \frac{\text{MeV}}{\Delta m} \right]^3$$



with:

$$b = 10 \cos^2 \theta_W / 3 \simeq 2.54$$

$$B^1 \rightarrow G^1 \gamma$$

$$b = 2 \cos^2 \theta_W \simeq 1.52$$

$$G^1 \rightarrow B^1 \gamma$$

$$b = 2 |N_{11}|^2$$

$$\chi \rightarrow \tilde{G} \gamma$$

$$b = |N_{11}|^2$$

$$\tilde{G} \rightarrow \chi \gamma$$

$$\chi = N_{11} (-i\tilde{\gamma}) + N_{12} (-i\tilde{Z}) + N_{13} \tilde{H}_u + N_{14} \tilde{H}_d$$



# Gravitational production

Particles are generically produced by a temporal depending geometry

- The number density is related to Bogoliubov coefficients  $\beta_k$

$$n = \frac{1}{2\pi^2 a^3} \int dk k^2 |\beta_k|^2$$

- Scalar mode wave functions in large  $k$  region:

$$\ddot{\phi}_k + 3H\dot{\phi}_k + \omega_k^2(a)\phi_k = 0$$

- WKB approximation:

$$\phi_k = \frac{1}{\sqrt{2\omega_k a^3}} \left( \alpha_{k,0} e^{-i\int \omega_k dt} + \beta_{k,0} e^{+i\int \omega_k dt} \right)$$

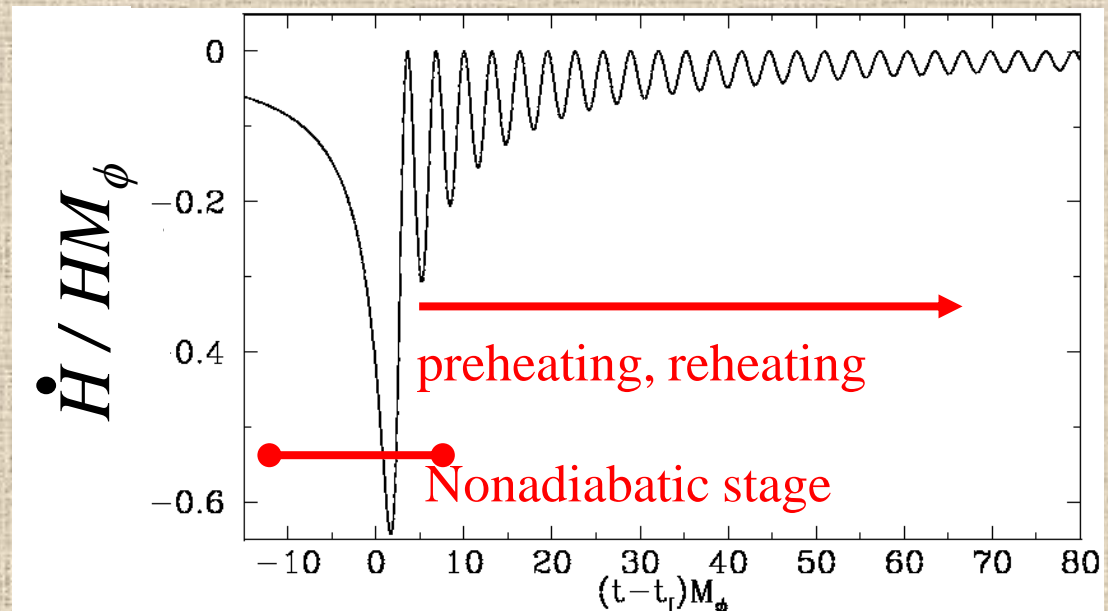
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Chaotic inflation, conformal coupling



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- Scaling of the DM density related to radiation:

$$\frac{\rho(t_0)}{\rho_R(t_0)} = \frac{\rho(t_{\text{RH}})}{\rho_R(t_{\text{RH}})} \left( \frac{T_{\text{RH}}}{T_0} \right)$$

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$$\frac{\rho(t_{\text{RH}})}{\rho_R(t_{\text{RH}})} \approx \frac{8\pi}{3} \frac{\rho(t_e)}{M_{\text{Pl}}^2 H^2(t_e)}$$

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- Abundance:  $\Omega \equiv \rho(t_0)/\rho_c(t_0)$  ←  $\rho_c(t_0) = 3H_0^2 M_{Pl}^2 / 8\pi$

$$\Omega h^2 \approx \Omega_R h^2 \frac{8\pi}{3} \left( \frac{T_{RH}}{T_0} \right) \frac{n(t_e) m}{M_{Pl}^2 H^2(t_e)}$$

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$$n = \frac{1}{2\pi^2 a^3} \int dk k^2 |\beta_k|^2$$

$$|\beta_k|^2 \approx \frac{\pi^2}{9} \exp\left(-4 \frac{(k/a_{\text{eff}}(r))^2 + m^2}{m\sqrt{H_{\text{eff}}^2(r) + R_{\text{eff}}(r)/6}}\right)$$

- Abundance:

$$\Omega h^2 \approx \Omega_R h^2 \frac{8\pi}{3} \left(\frac{T_{\text{RH}}}{T_0}\right) \frac{n(t_e)m}{M_{\text{Pl}}^2 H^2(t_e)}$$

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$$n = \frac{1}{2\pi^2 a^3} \int dk k^2 |\beta_k|^2$$

- Abundance:

$$\Omega_X h^2 \approx \left( \frac{M_X}{10^{11} \text{ GeV}} \right)^2 \frac{T_{RH}}{10^9 \text{ GeV}} \left( \frac{M_X}{H_e} \right)^{1/2} \exp(-2M_X / H_e)$$

 Supermassive DM:  $M_X > 10^9 \text{ GeV}$

Chung, Crotty, Kolb, Riotto, arXiv:hep-ph/0104100

# Relic Density from symmetry breaking

- **Non-topological and topological solitons are generically formed:**
  - Strings: Non trivial first homotopy group
  - Monopoles: Non trivial second homotopy group
  - Skyrmions: Non trivial third homotopy group
- **They can be classified according to the homotopy of the coset space of the symmetry breaking pattern:**

$G$



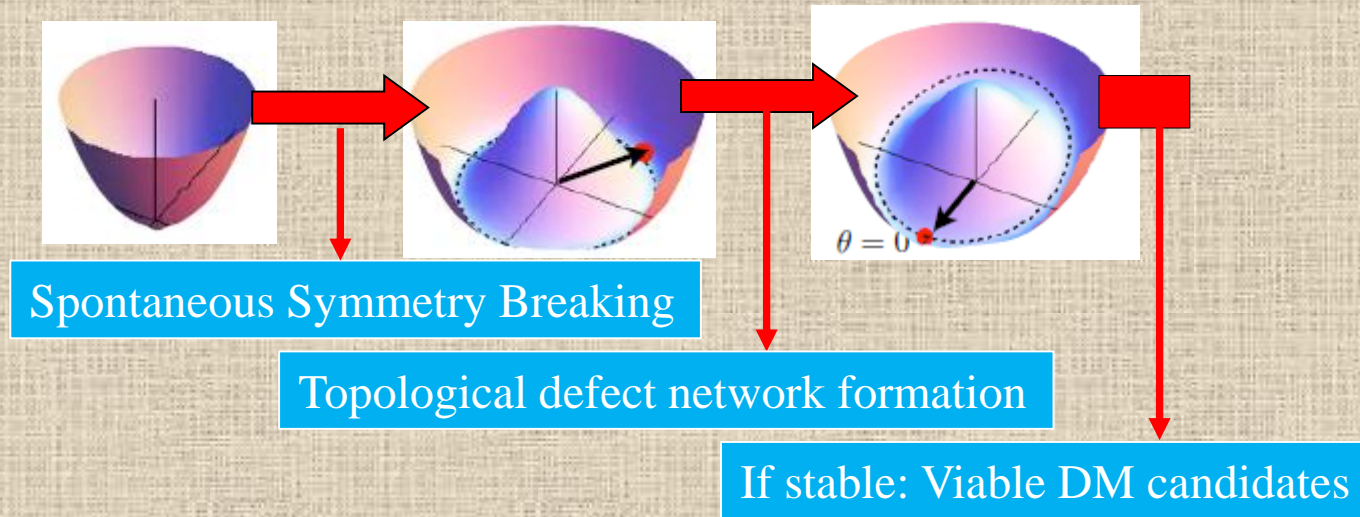
$H$

■ Coset space:  $G/H$



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# Particle defects: Monopoles

- **Kibble mechanism:**

- Correlation length  $\xi$
- Formation of one monopole per Hubble volume

$$n_M \sim \xi^{-3} \Rightarrow n_M \simeq H^3$$

- Universe dominated by radiation:

$$H = \sqrt{\frac{4\pi^3 g_*}{45}} \frac{T^2}{M_P} = 1.66\sqrt{g_*} \frac{T^2}{M_P}$$

$$s = \frac{2\pi^2}{45} g_{*s} T^3$$

- Number density of monopoles:

$$\frac{n_M}{s} \simeq \sqrt{\frac{16\pi^5}{45}} \frac{g_*^{3/2}}{g_{*s}} \left(\frac{T_c}{M_P}\right)^3 = 107 \left(\frac{T_c}{M_P}\right)^3 \leftarrow$$

# Particle defects: Monopoles

## ■ Kibble-Zurek mechanism:

- The transition takes place in a finite time

- Correlation length

- Relaxation time

$$\begin{cases} \xi = \xi_0 |\epsilon|^{-\nu} \\ \tau = \tau_0 |\epsilon|^{-\mu} \end{cases}$$

$$\epsilon \equiv \frac{T_c - T}{T_c}.$$

$$\begin{cases} t - t_c = \tau_Q \epsilon(t) \\ \tau(t_*) = t_* - t_c \end{cases} \Rightarrow \xi(t_*) = \xi_0 \left( \frac{\tau_Q}{\tau_0} \right)^{\frac{\nu}{\mu+1}}$$

- Quenching time:

$$\tau_Q \xrightarrow{t \rightarrow t_c} 2t_c = H^{-1}(T_c)$$

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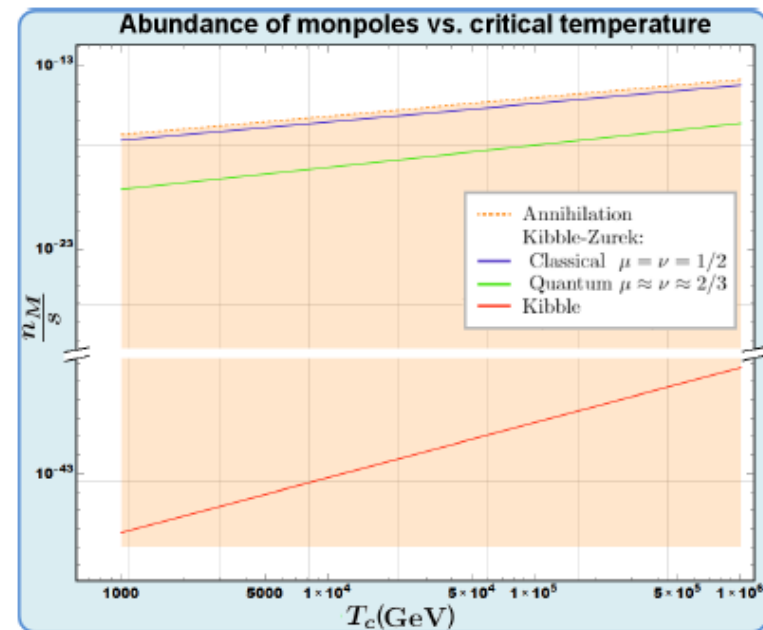
$$\frac{n_M}{s} \simeq 0.007 \left( \frac{24 T_c}{M_P} \right)^{\frac{3\nu}{\mu+1}} \leftarrow$$

- Landau-Ginzburg:  $(\nu = 1/2)$

$$\frac{n_M}{s} \simeq 0.1 \left( \frac{T_c}{M_P} \right) \leftarrow$$

- Fiducial:  $(\nu = 2/3)$

$$\frac{n_M}{s} \simeq 0.35 \left( \frac{T_c}{M_P} \right)^{\frac{6}{5}} \leftarrow$$



# Particle defects: Monopoles

## ■ Kibble-Zurek mechanism:

■ Correlation length

■ Relaxation time

$$\begin{cases} \xi = \xi_0 |\epsilon|^{-\nu} \\ \tau = \tau_0 |\epsilon|^{-\mu} \end{cases}$$

$$\epsilon \equiv \frac{T_c - T}{T_c}.$$

$$\Omega_M h^2 \sim 2 \cdot 10^{12} \left( \frac{1.97 \cdot 10^{-12}}{x_c} \right)^{\frac{3\nu}{\mu+1}} \left( \frac{m_M}{\text{PeV}} \right)^{\frac{3\nu}{\mu+1} + 1}$$

$$x_c = m_M / T_c$$

■ Landau-Ginzburg:

$$\Omega_M h^2 \sim \frac{2.30}{x_c} \left( \frac{m_M}{\text{PeV}} \right)^2$$

■ Fiducial model:

$$\Omega_M h^2 \sim \frac{1.9 \cdot 10^{-2}}{x_c^{6/5}} \left( \frac{m_M}{\text{PeV}} \right)^{\frac{11}{5}}$$

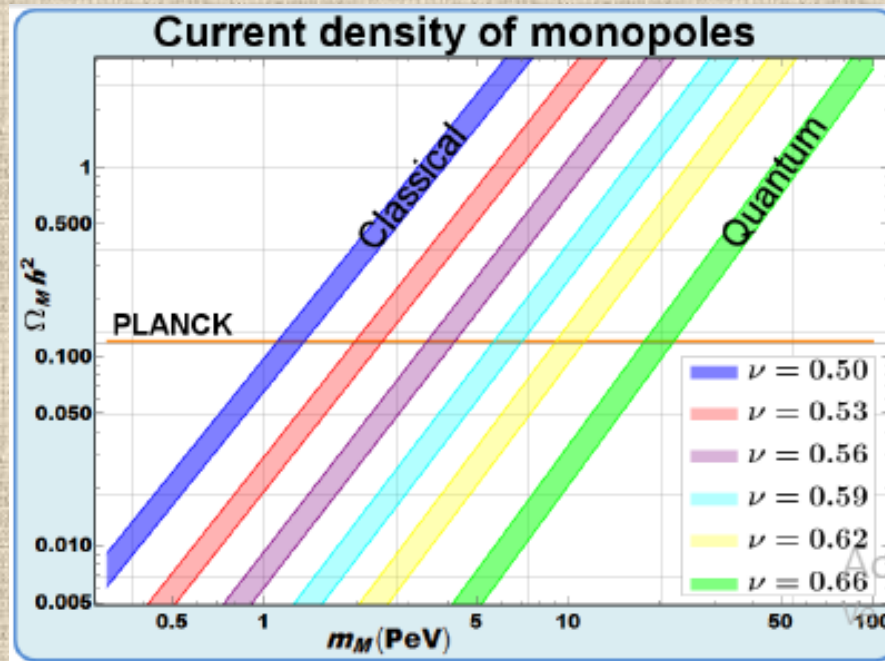
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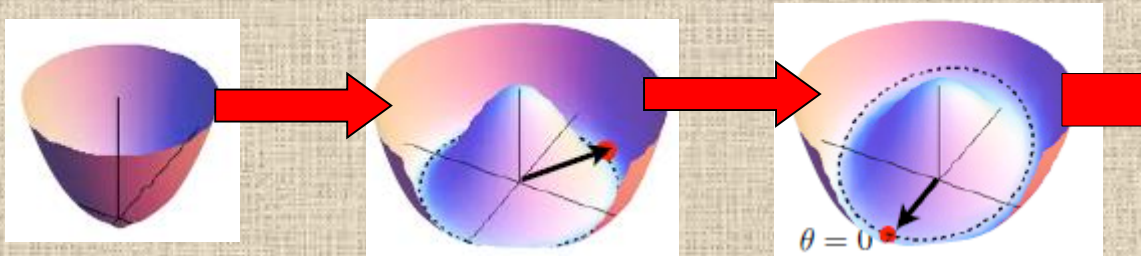
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# Relic Density from symmetry breaking

- **Non-topological and topological solitons are generically formed:**

- Strings: Non trivial first homotopy group
- Monopoles: Non trivial second homotopy group
- Skyrmions: Non trivial third homotopy group



- **Monopoles:**

$$\Omega_{PD} h^2 \approx 1.5 \times 10^9 \left( \frac{x_c T_c}{1 \text{TeV}} \right) \left( \frac{30 T_c}{M_{pl}} \right)^{\frac{3\nu}{1+\nu}}$$

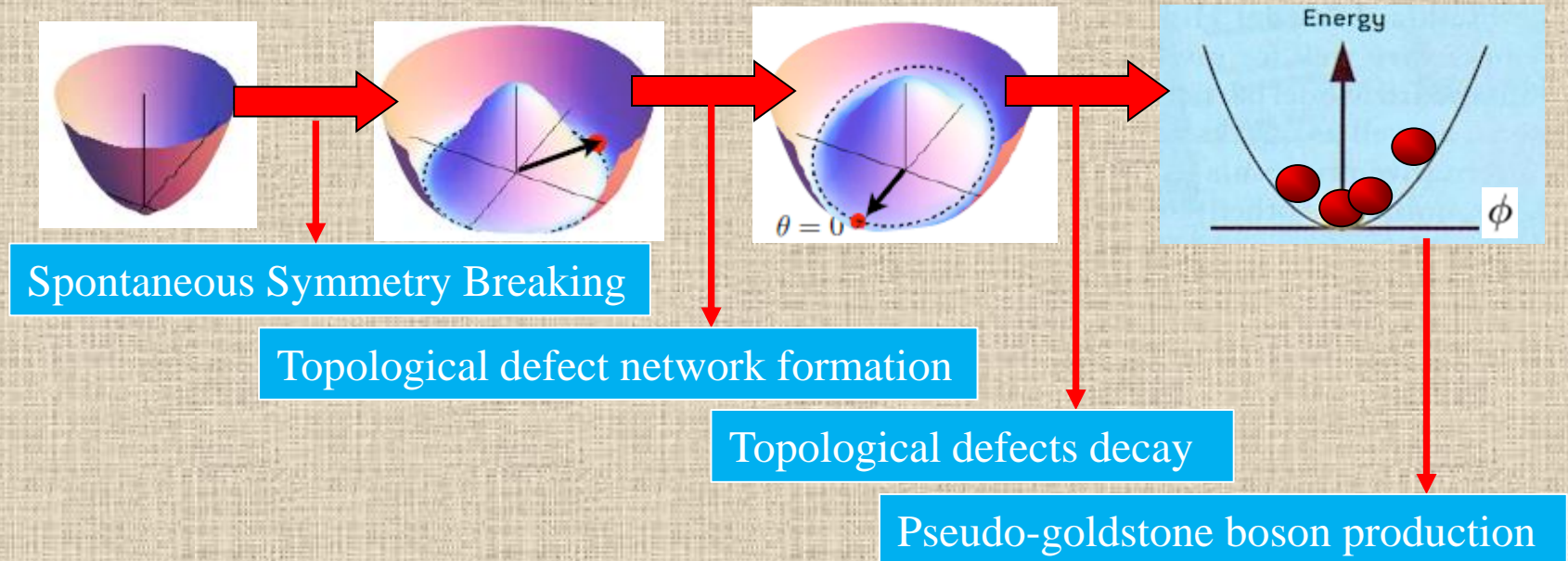
$$T_c \approx 10^6 \text{ GeV}$$

Murayama and Shu, arXiv:0905.1720v1

*Lectures on Dark Matter*  
*Jose A. R. Cembranos*

# Relic Density from symmetry breaking

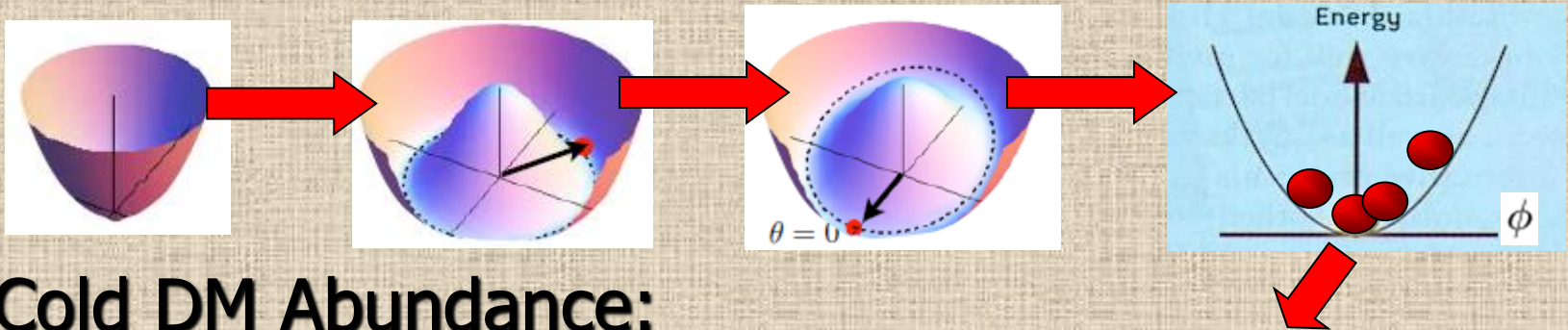
- **Non-topological and topological solitons are generically formed:**
  - Strings: Non trivial first homotopy group
  - Monopoles: Non trivial second homotopy group
  - Textures or skyrmions: Non trivial third homotopy group





# Relic Density from symmetry breaking

- **Topological defects are generically formed depending on the symmetry pattern:**
  - Strings: Non trivial first homotopy group
  - Monopoles: Non trivial second homotopy group
  - Textures or skyrmions: Non trivial third homotopy group



**Cold DM Abundance:**

$$\Omega_{\phi} h^2 \simeq 0.15 \left[ \frac{m_s}{1 \text{ eV}} \right]^{\frac{1}{2}} \left[ \frac{\phi_1}{10^{12} \text{ GeV}} \right]^2 \left[ \frac{1}{(\gamma_{s1} g_{s1})^4} \right]^{\frac{1}{4}}$$

# Relic Density from Misalignments

- Bosonic particles may have important abundance due to initial displacements.

- Misalignment mechanism

- For  $H(T) \gg m_s$   $\longrightarrow \phi = \phi_1$

$$T_1 \simeq 15.5 \text{ TeV} \left[ \frac{m_s}{1 \text{ eV}} \right]^{\frac{1}{2}} \left[ \frac{100}{g_{e1}} \right]^{\frac{1}{4}}$$

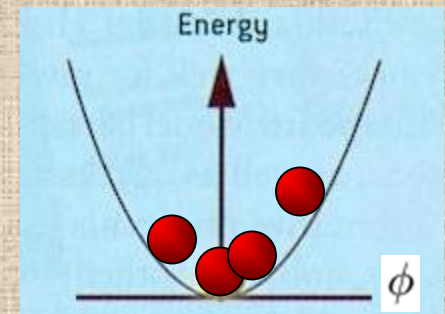
- For  $3H(T) \leq m_s$   $\longrightarrow \phi$  oscillates around the minimum of its potential. These oscillations correspond to a zero-momentum condensate.

- Number density:

$$n_\phi \sim m_0 \phi_1^2 / 2$$

- Abundance:

$$\Omega_\phi h^2 \simeq \frac{(n_\phi/s)(s_0/\gamma_{s1})}{\rho_{\text{crit}}} m_0$$

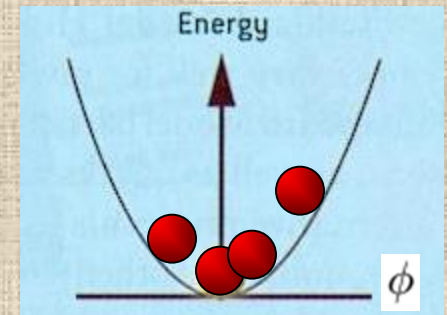


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- It oscillates from:  $T_1 \simeq 15.5 \text{ TeV} \left[ \frac{m_s}{1 \text{ eV}} \right]^{\frac{1}{2}} \left[ \frac{100}{g_{e1}} \right]^{\frac{1}{4}}$

## Cold DM Abundance:

$$\Omega_\phi h^2 \simeq 0.86 \left[ \frac{m_s}{1 \text{ eV}} \right]^{\frac{1}{2}} \left[ \frac{\phi_1}{10^{12} \text{ GeV}} \right]^2 \left[ \frac{100 g_{e1}^3}{(\gamma_{s1} g_{s1})^4} \right]^{\frac{1}{4}}$$

Cembranos, PRL102:141301 (2009)

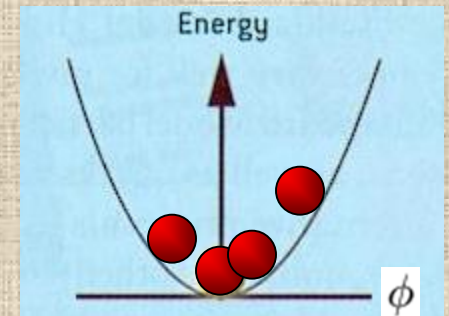
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- For the QCD axion:

$$m_0 \simeq \Lambda_{\text{QCD}}^2 / f_a \simeq 0.6 \times 10^{-4} \text{ eV} \left( \frac{10^{11} \text{ GeV}}{f_a} \right)$$

$$\phi_1 \equiv \theta_1 f_a$$

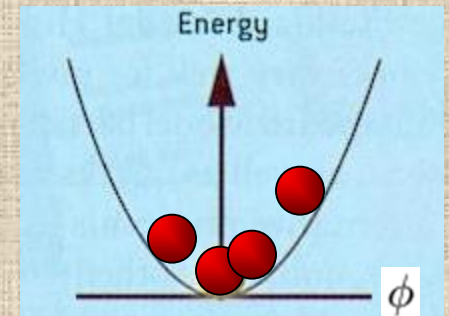
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$$m(T) \simeq 0.1 m_0 \left[ \frac{100 \text{ MeV}}{T} \right]^{3.7}$$

# *Asymmetric DM*

The abundance of DM may be related to a different number of DM particles versus DM antiparticles.

- A Dark global symmetry can be postulated associated with a dark baryonic number:
  - ▶  $B_V$  is broken, whereas  $B_D$  is not.
  - ▶  $B_D$  is broken, whereas  $B_V$  is not.
  - ▶  $B_V$  and  $B_D$  are both broken.
  - ▶ A linear combination of  $B_V$  and  $B_D$  can be broken:  $X$
- Different possibilities for production:

# *Asymmetric DM*

The abundance of DM may be related to a different number of DM particles versus DM antiparticles.

- **A Dark global symmetry can be postulated associated with a dark baryonic number:**
- **Different possibilities for production:**
  - ▶ Asymmetric freeze-out
  - ▶ Asymmetric freeze-in
  - ▶ Violating  $X$  and CP Decaying DM
  - ▶ Coherent bosonic background violating  $X$  and CP (Affleck-Dine mechanism)
  - ▶ First order phase transition ( $X$  is violated through sphalerons)
  - ▶ Spontaneous genesis (CPT violation)

# Conclusions

**We have discussed about different DM production mechanisms:**

- ▶ **Freeze-out mechanism**
- ▶ **Freeze-in mechanism**
- ▶ **Decays of other particles**
- ▶ **Gravitational production**
- ▶ **Misalignment mechanism**
- ▶ **Spontaneous symmetry breaking**
- ▶ **Asymmetric DM**



# Conclusions

## Examination sheet

Jose A. R. Cembranos \*

*Departamento de Física Teórica I, Universidad Complutense de Madrid, E-28040 Madrid, Spain.*

I.- Estimate the maximum and minimum mass and cross section interaction for the dark matter candidate in order to account for the dark matter abundance of the standard model of cosmology depending on the production mechanism:

- I.a.- Freeze-out mechanism
- I.b.- Freeze-in mechanism
- I.c.- Decays of other particles
- I.d.- Gravitational production
- I.e.- Misalignment mechanism
- I.f.- Spontaneous symmetry breaking:
  - I.f.1.- Topological Defects
  - I.f.2.- Pseudo-Nambu-Goldstone bosons
- I.g.- Asymmetric DM

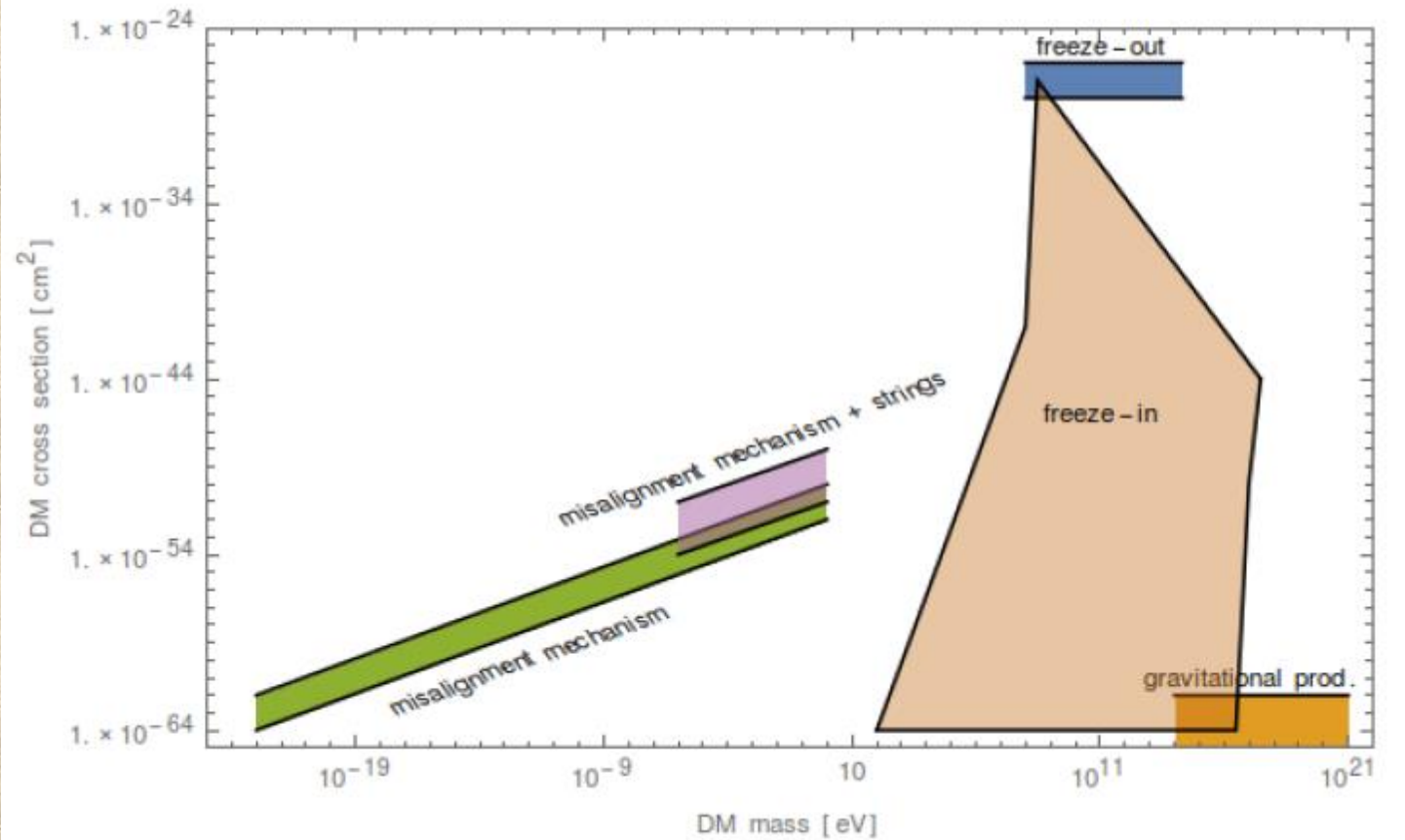
Comment your assumptions and plot the final results in a common figure  $\sigma$  vs  $m$  (for example,  $\sigma$  can be defined as  $\sigma \equiv \langle \sigma_A v \rangle / \sqrt{\langle v^2 \rangle}$  for the freeze-out mechanism, but other definitions can be used for other types of production).

DM School, Lund University (2016)

# Conclusions

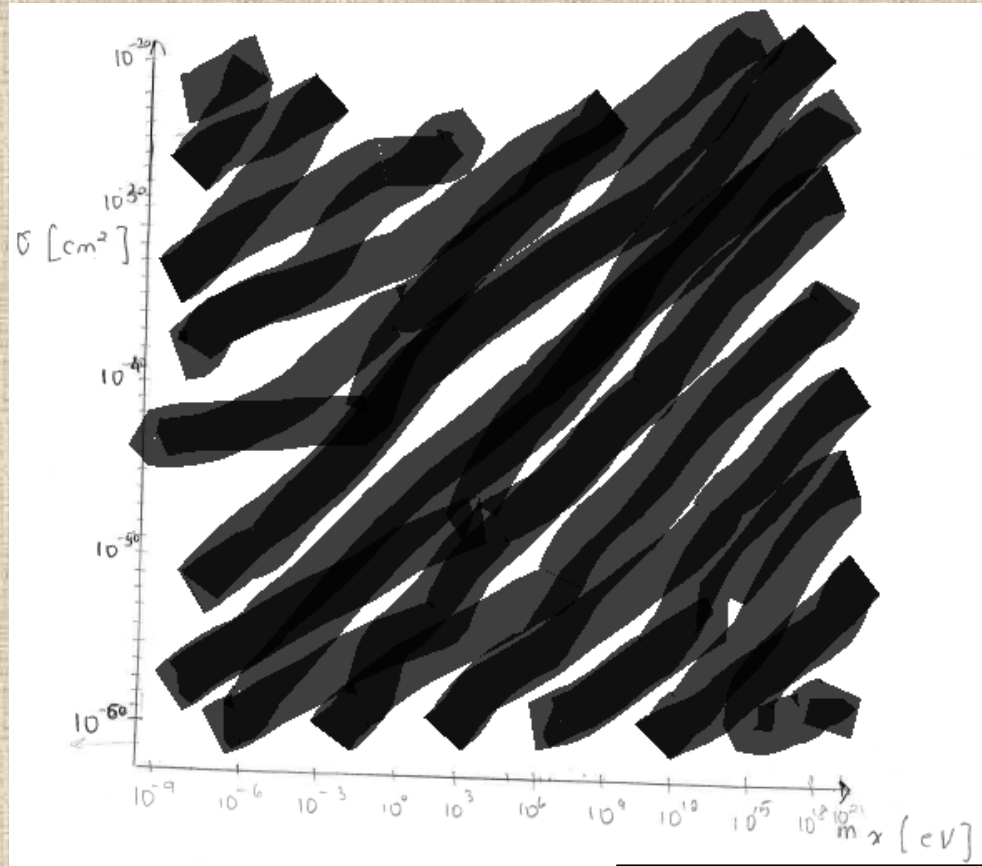
DM School Lund - Cembranos

Sebastian Baum - October 3, 2016



DM School, Lund University (2016)

# Conclusions



DM School, Lund University (2016)

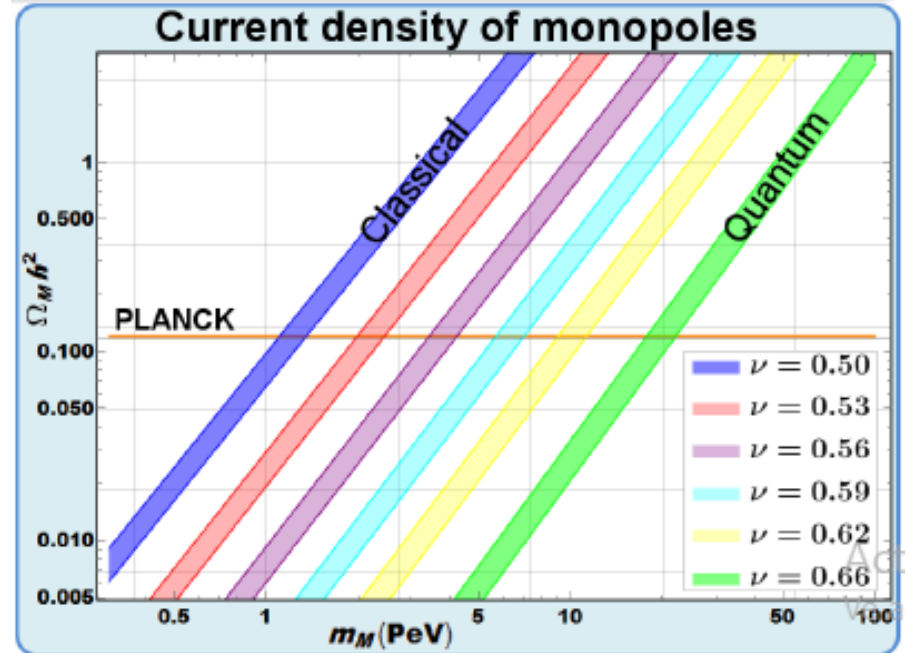
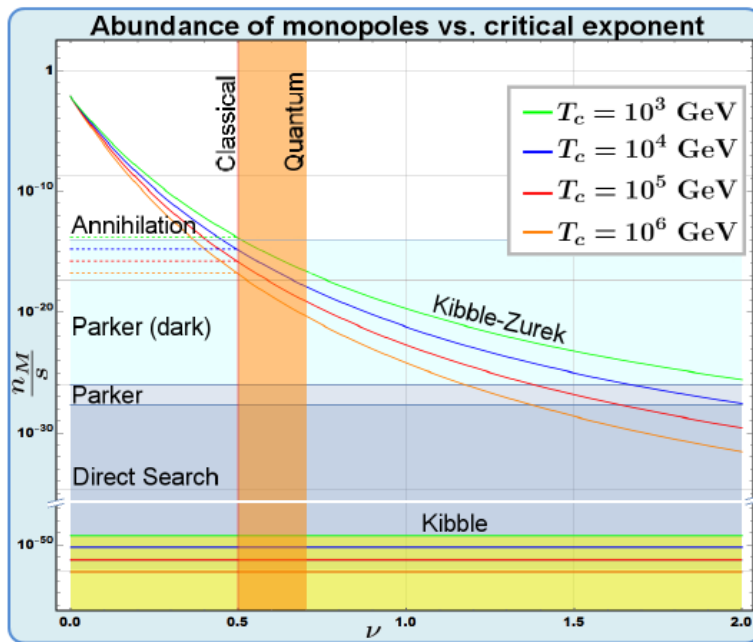
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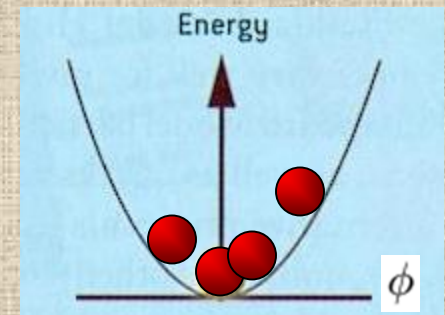
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Cembranos, PRL102:141301 (2009)