Supermultiplets in $\mathcal{N} = 1$ SU(2) SUSY Yang-Mills Theory

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Münster-DESY(-Regensburg-Jena) collaboration

GRK Retreat 2017 - Marienheide

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Graduiertenkolleg 2149 Research Training Group

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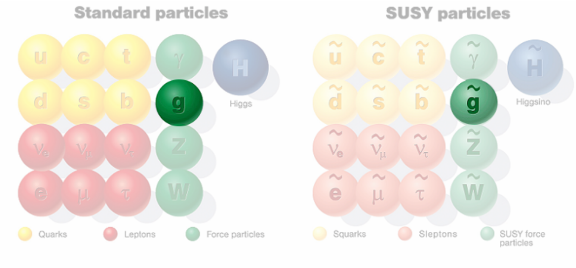
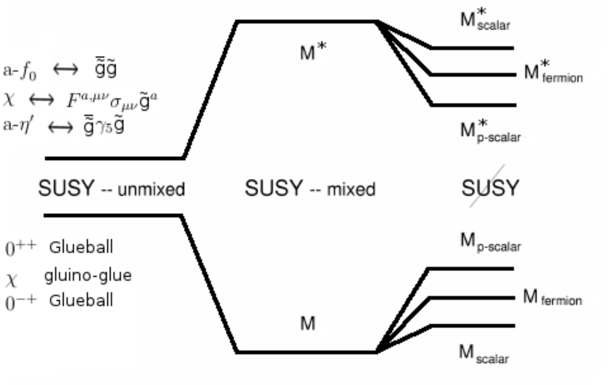


- $\mathcal{L} = \frac{1}{2}\mathrm{Tr}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\bar{\tilde{g}}\not{D}\tilde{g} - \frac{m_g}{2}\bar{\tilde{g}}\tilde{g}\right)$
- $A_\mu^a(x)$: gauge fields
- $\tilde{g}(x)$: gluino fields, Majorana fermions in adjoint representation
  - $(D_\mu\tilde{g})^a = \partial_\mu\tilde{g}^a + gf_{abc}A_\mu^b\tilde{g}^c$, here: $f_{abc} = \epsilon_{abc}$
- $\frac{m_g}{2}\bar{\tilde{g}}\tilde{g}$: soft SUSY-breaking term



G. R. Farrar, G. Gabadadze, M. Schwetz, “, Phys. Rev. D60 (1999) 035002, arXiv:hep-th/9806204.

Expectation values in lattice field theory

- Interested in $\langle O \rangle = \frac{\int \mathcal{D}[\Phi,\Psi]\, e^{-S_E[\Phi]} O[\Phi,\Psi]}{\int \mathcal{D}[\Phi,\Psi] e^{-S_E[\Phi,\Psi]}}$
- Integrate out the fermions
  $\Rightarrow Z = \int \mathcal{D}[\Phi,\Psi]\, e^{-S_G[\Phi]} e^{-S_F[\Phi,\Psi]} = \int \mathcal{D}[\Phi]\, e^{-S_G[\Phi]} \det Q$
  - $\det Q = \int \mathcal{D}[\Psi]\, e^{-\bar{\Psi} Q \Psi}$: Fermion determinant
- For different lattice spacings a and (bare) gluino-masses m_g generate ensembles of lattice gauge configurations with probability distribution $e^{-S_E[\Phi]} \det Q$
  - Hybrid Monte Carlo (PHMC)
- Calculate $\langle O \rangle$ on the ensembles
- for each a perform chiral extrapolation $m_\pi^2 \rightarrow 0$
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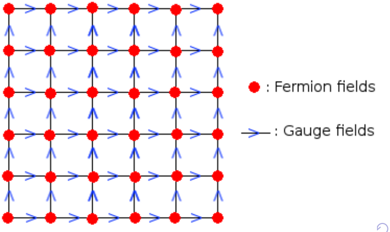

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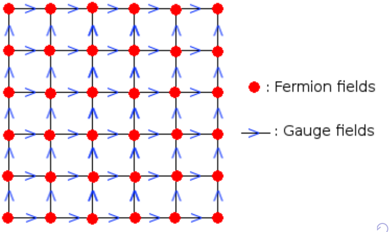

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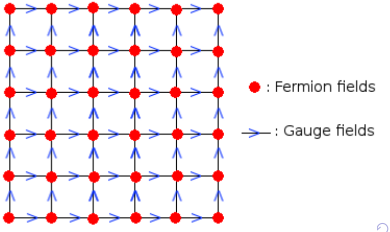

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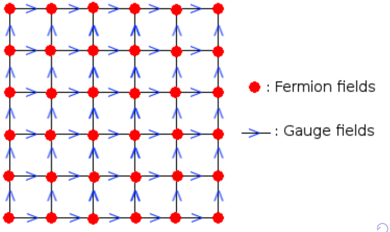

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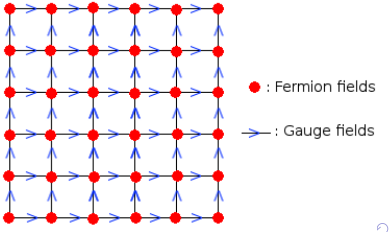

Extracting masses from Correlation Function

- Correlation function
  $\tilde{C}(x,y) = \left\langle O(x) O^{\dagger}(y) \right\rangle$
  - Operator O is interpolating field, e.g. $O(x) = \bar{\tilde{g}}(x)\tilde{g}(x)$
- Translation invariance + 0-momentum projection leads to
  $C(\Delta t) = N \sum_{\vec{x}} \left\langle O(\Delta t) O^{\dagger}(0) \right\rangle$

a-η'- glueball
@$\beta = 1.9$, $\kappa = 0.14435$

- Spectral decomposition:
  $C(\Delta t) = a_0^2 + \sum_{n=1}^{\infty} a_n^2 \mathrm{e}^{-E_n \Delta t} \pm \mathrm{e}^{-E_n (T - \Delta t)}$

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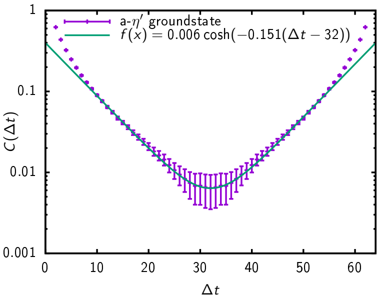

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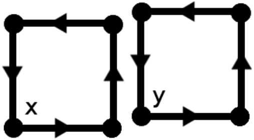
Particles and their operators

- Glueballs: $O(x) = \sum_{i<j} P_{ij}(x)$



- Gluino-glue: $O^{\alpha}(x) = \sum_{i<j} \sigma_{ij}^{\alpha\beta} Tr_c \left[P_{ij}(x)\tilde{g}^{\beta}(x)\right]$

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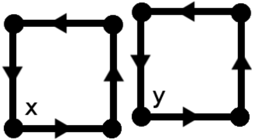

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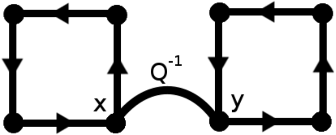

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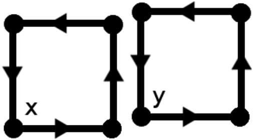

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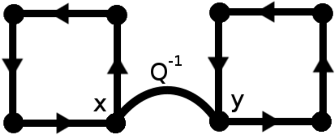

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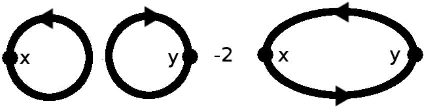

Smearing

- Improve signal-to-noise ratio by applying smearing techniques:
- For gluonic operators: APE Smearing

$$U'_\mu(x) = U_\mu + \epsilon_{\text{APE}} \sum_{\nu=\pm1,\nu\neq\mu}^{\pm3} U^\dagger_\nu(x+\mu)U_\mu(x+\hat{\nu})U_\nu(x)$$

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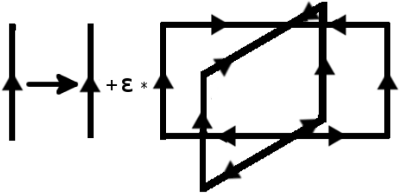

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$\tilde{g}'(\vec{x}, t) = \sum_{\vec{y}} F(\vec{x}, \vec{y}) \tilde{g}(\vec{y}, t)$

- $F(\vec{x}, \vec{y})$ =Iterative solution of 3d Klein-Gordon equation for source and sinks :

$$F_{\beta b, \alpha a}(\vec{x}, \vec{y}) = \delta_{\vec{x}, \vec{y}} \delta_{\beta\alpha} + \delta_{\beta\alpha} \sum_{i=1}^{N_J} \left(\kappa_J \sum_{\mu=1}^{3} \left[\delta_{\vec{y}, \vec{x}+\mu} U_{\mu, ba}(x) + \delta_{\vec{y}+\hat{\mu}} U^{\dagger}_{\mu, ba}(x) \right] \right)^i$$

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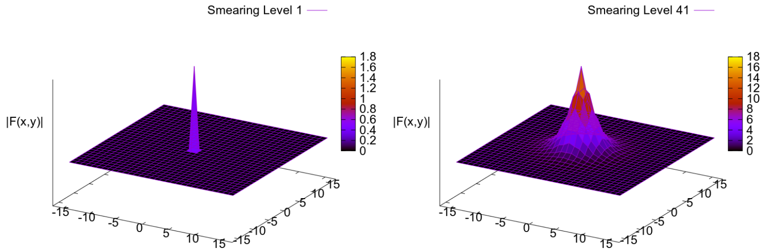

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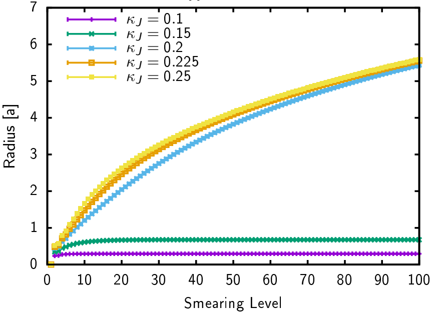
Jacobi Smearing

- Smearing Radius:

$$R_J = \frac{\sum_{\vec{x}} |\vec{x}|^2 |F(\vec{x},0)|^2}{\sum_{\vec{x}} |F(\vec{x},0)|^2}$$



$\Rightarrow$Use $\kappa_{\text{Jacobi}} = 0.2$
$\Rightarrow$Use smearing levels up to 80
$\Rightarrow$Optimizing the signal lead to choosing smearing levels 0, 40 and 80

APE smearing:

- similar analysis as for Jacobi smearing
- $\epsilon_{\text{APE}} \sim 0.4$
- smearing levels up to 95 for $32^3 \times 64$ lattice
- i.e. {5,15,...,95} for gluino-glue

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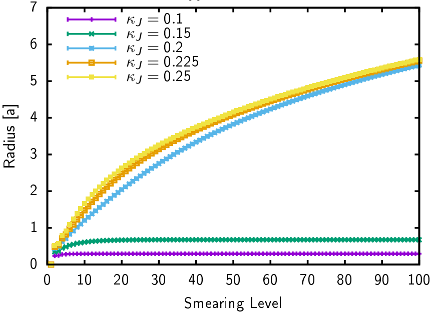

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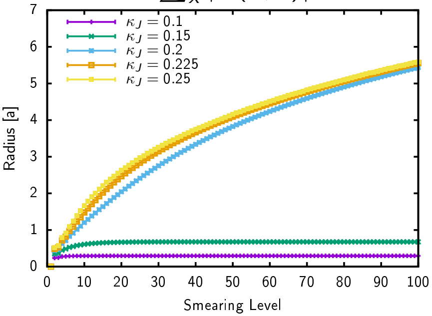

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Variational method

- Correlation Matrix $C_{ij}(\Delta t) = \left\langle O_i(\Delta t) O_j^\dagger(0) \right\rangle$
- Generalized Eigenvalue Problem (GEVP): $C(t)\vec{v}^{(n)} = \lambda^{(n)}(t, t_0) C(t_0) \vec{v}^{(n)}$
- $\lim_{t\to\infty} \lambda^{(n)}(t, t_0) \propto \mathrm{e}^{-m_n(t-t_0)} \left(1 + \mathcal{O}\left(\mathrm{e}^{-\Delta m_n (t-t_0)}\right)\right)$
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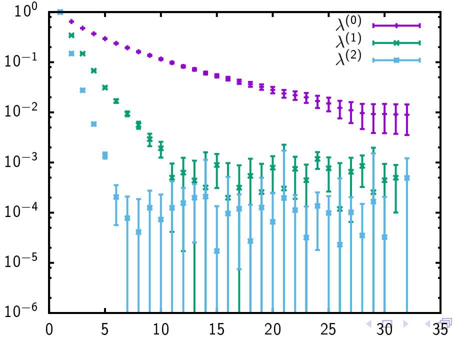

Exploiting supersymmetry in GEVP

- Mixing of meson and glueball states allows using a larger operator basis for determining the masses in the $0^{++}$and 0^{-+} channel
  - e.g. a-f_0 and 0^{++}-glueball
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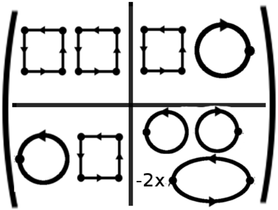
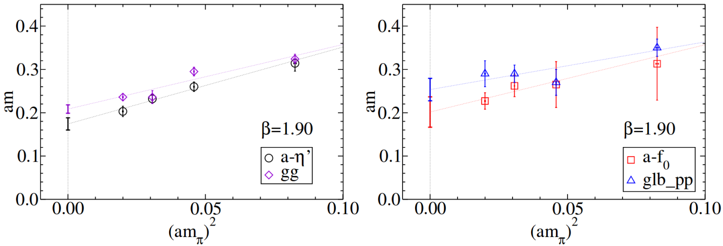


Simulation

- Configurations produced at Jülich Supercomputing Centre
- Measurements performed on
  - local HPC clusters in Münster
    - PALMA, NWZPHI (Xeon-Phi)
  - Jülich Supercomputing Centre

Groundstate multiplet



Extrapolation of the lightest supermultiplet to the chiral limit. Desy-Münster collaboration arXiv:1512.07014

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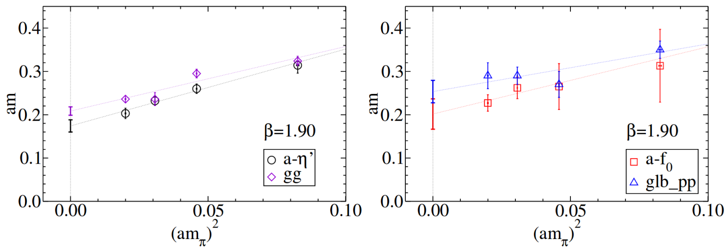

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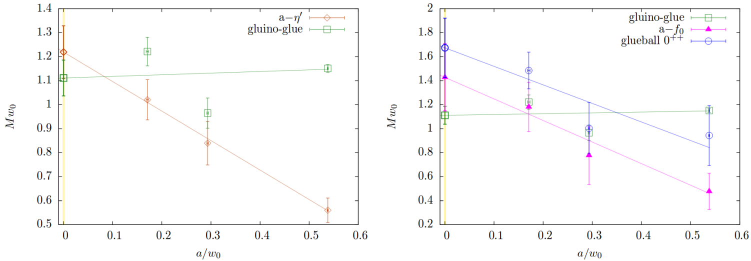
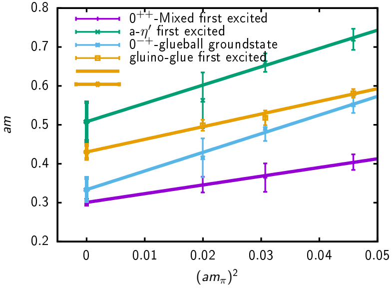


Extrapolation of the lightest supermultiplet to the continuum limit. Desy-Münster collaboration JHEP 03 (2016) 080

Excited Multiplet @ $\beta = 1.9$, preliminary



Linear chiral extrapolation at $\beta = 1.9$

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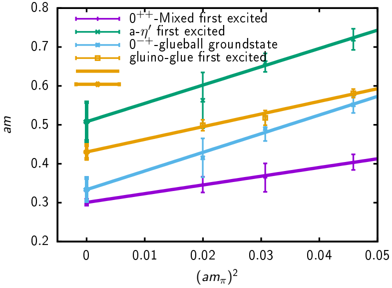

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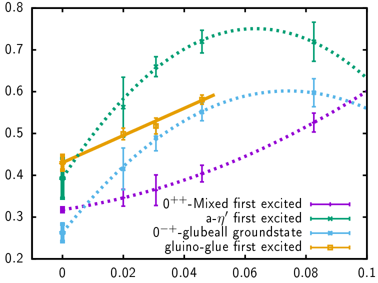
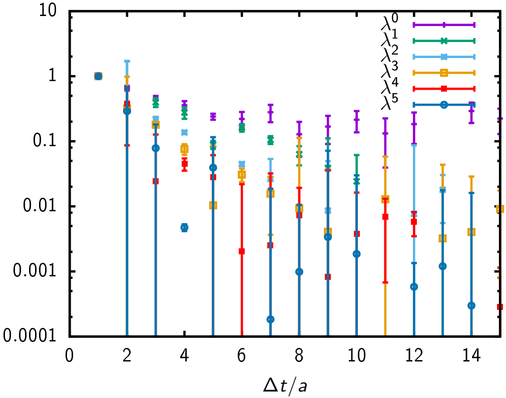


Quadratic chiral extrapolation at $\beta = 1.9$

Challenges

- Eigenvalues need to be ordered correctly



Generalized eigenvalues of 0^{++}-channel at $\beta = 1.9$, $\kappa = 0.1433$ sorted by value

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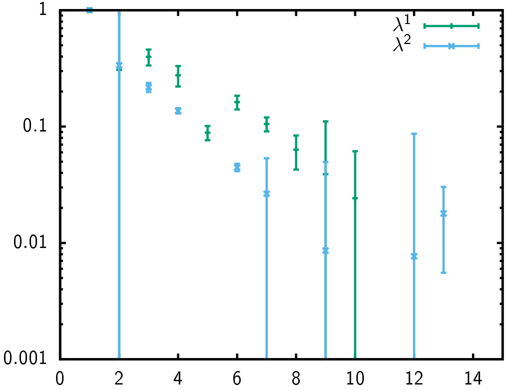

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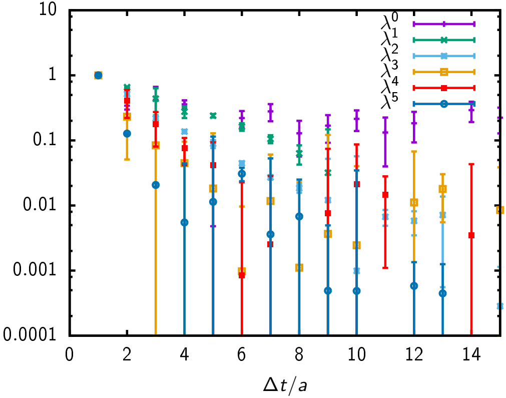

Generalized eigenvalues of 0^{++}-channel at $\beta = 1.9$, $\kappa = 0.1433$ sorted by new method

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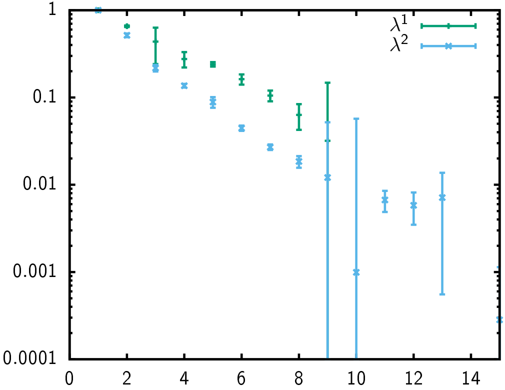

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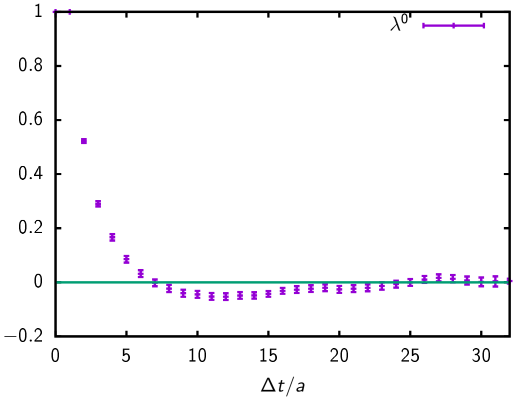
Challenges

- Eigenvalues need to be ordered correctly
- Eigenvalues don't always behave as expected
  - large autocorrelation



0^{-+}- glueball $@\beta = 1.9,\ \kappa = 0.14435$

Challenges

- Eigenvalues need to be ordered correctly
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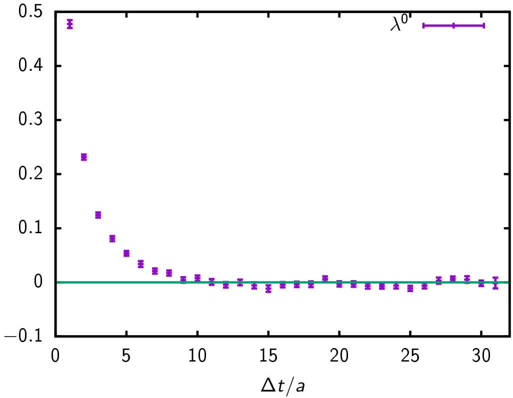
  - large autocorrelation
  - use "Derivative trick": $\Rightarrow$ Use $\tilde{C}(t) = C(t) - C(t+1)$"



0^{-+}- glueball @$\beta = 1.9$, $\kappa = 0.14435$

Things to be checked

- Second order polynomial fit fits the data too well
  - errorbars overestimated?
- Within errors there seems so be no mixing in the 0^{-+}-channel
  - off-diagonal entries of the correlation matrix are zero within errors
- 0^{-+}-glueball and $a - \eta'$ masses differ

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Summary & Outlook

- Agreement with the predicted supermultiplet structure for the groundstate
  $\Rightarrow$ SUSY is restored in the continuum limit
- 0^{-+}-glueball groundstate heavier than groundstate multiplet
  $\Rightarrow$might become lighter in the continuum limit
  $\Rightarrow$maybe it belongs to the first excited multiplet
- Work to be done:
  - Continuum extrapolation of the excited multiplet
  - reanalysis of groundstate multiplet using the full correlation matrix
  - Determine mixing between glueball and mesonic states using the variational method:
    - learn about mixing in the ~~SUSY~~-phase

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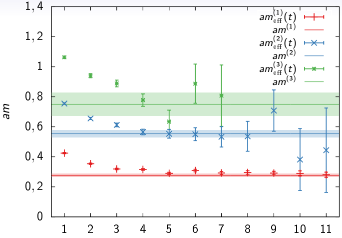
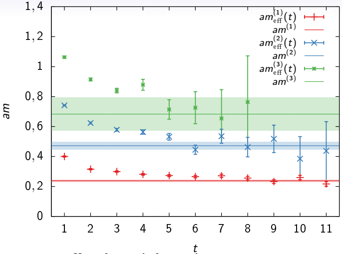
Thank you for your attention!!

Westfälische Wilhelms-Universität Münster

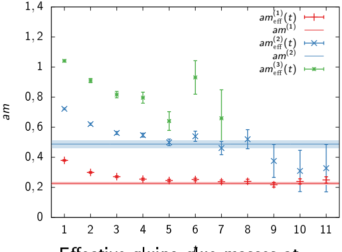
Graduiertenkolleg 2149 Research Training Group



Effective gluino-glue masses at $\kappa = 0.14387$



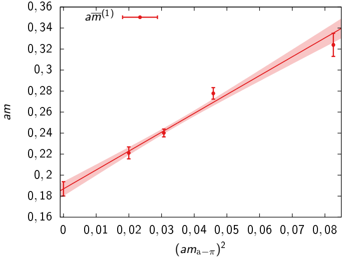
Effective gluino-glue masses at $\kappa = 0.14415$



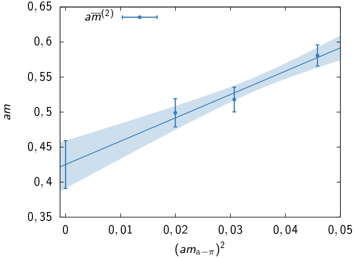
Effective gluino-glue masses at $\kappa = 0.14435$

S. Kuberski, Masterthesis, Uni Münster

- $\kappa = \frac{1}{2am+8}$
- $\beta = \frac{2N}{g^2}$
- $m_{\text{eff}} = \ln \frac{C(\Delta t)}{C(\Delta t+1)}$



Extrapolation to the chiral limit (groundstate)



Extrapolation to the chiral limit (first excited state)

S. Kuberski, Masterthesis, Uni Münster

- Calculate the lowest eigenvalues of Q and corresponding eigenvectors
  - using Arnoldi (ARPACK)
  - Chebyshev Polynomials of order 11
  - Even/Odd-Preconditioning
- Stochastic estimator technique for space orthogonal to the previously calculated eigenvectors:
  - $\frac{1}{N_S}\sum_i^{N_S} \left|\eta^i\right\rangle \left\langle\eta^i\right| = \mathbb{1} + \mathcal{O}\left(\sqrt{N_S}\right)$
    - use $\mathbb{Z}_4$-noise
  - $Q\left|s^i\right\rangle = \left|\eta^i\right\rangle$
  - $Q^{-1} = \frac{1}{N_S}\sum_i^{N_S} \left|s^i\right\rangle \left\langle\eta^i\right|$
  - Conjugate gradient
  - $N_S = 40$ for $\beta = 1.9$, $32^3 \times 64$